

Formal Logic for Informal Logicians*

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Abstract: Classical logic yields counterintuitive results for numerous propositional argument forms. The usual alternatives (modal logic, relevance logic, etc.) generate counterintuitive results of their own. The counterintuitive results create problems—especially pedagogical problems—for informal logicians who wish to use formal logic to analyze ordinary argumentation. This paper presents a system, PL₋ (propositional logic minus the funny business), based on the idea that paradigmatic valid argument forms arise from justificatory or explanatory discourse. PL₋ avoids the pedagogical difficulties without sacrificing insight into argument

Résumé: Les logiques classiques et alternatives (ex. modale) appliquées aux formes propositionnelles des arguments mènent à des résultats problématiques qui créent des ennuis—en particulier, pédagogiques—pour les logiciens non formels qui désirent employer ces logiques pour analyser des arguments courants. On présente un système de logique propositionnelle fondé sur l'idée que les formes des arguments valides paradigmatiques proviennent des discours justificatifs ou explicatifs. Cette logique évite les difficultés pédagogiques sans sacrifier ses éclaircissements de la structure des arguments.

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1. Introduction

Informal logicians study discursive practice in order to articulate principles and techniques implicit in that practice. Some informal logicians are stridently anti-formalist. For example, Paul 1985 complains about the “formulated-but-unlived” character of logical systems (15). Anyone who has taught classical logic can sympathize with Paul’s complaint; the material conditional and its attendant paradoxes go a long way toward undermining respect for the discipline of logic. But other informal logicians accept formal logic as a valuable tool for argument identification and evaluation (e.g., Hatcher 1999); it can be useful for discerning a simple structure in a complicated argument. I count myself in the latter group. But experience also leads me to take seriously Paul’s complaint about formulated-but-unlived logical systems. I find consistently that students—especially good ones—

feel a tension between discursive practice and various moves sanctioned by classical logic. In order to overcome this tension, I propose a formal system, PL– (propositional logic minus the funny business), which avoids the impression that logic is just a symbol game without sacrificing insight into argument structure.

Most formal logics aim to represent faithfully our intuitions about both valid and invalid instances of arguments. It is well-known that classical propositional logic fails to achieve this goal. For example, (1)

(1) I'm in Arizona

∴ If I'm in New York, I'm in Arizona,

is valid in classical propositional logic and yet intuitively invalid. It is one of the so-called paradoxes of material implication. These paradoxes generate a family of propositional inferences that are classically valid but intuitively invalid.¹ Of course, the past century has seen numerous and varied attempts to avoid the paradoxes of material implication. Lewis's strict implication aimed to solve the paradoxes by adding a modal operator to classical logic (Lewis 1918). Nelson's logic of intensional relations (Nelson 1930), Parry's analytic implication (Parry 1989) and then Anderson and Belnap's relevance logic (Anderson and Belnap 1975) all attempted to improve on Lewis's system by developing *alternatives* to classical propositional logic. Today there is an empire of non-classical logics so vast that no one has traversed them all. Hence, it is not feasible to discuss piecemeal the suitability of the non-classical alternatives for the purposes of informal logic. Yet there are general considerations which show that the non-classical logics are not what is wanted for the purpose of analyzing ordinary argumentation.

In the first place, many of the non-classical logics have counter-intuitive results of their own. Thus, Lewis's strict implication involves the dubious thesis that a necessary truth is entailed by any statement whatsoever. Dissatisfaction with the Lewis paradoxes motivates relevance logic. But relevance logicians reject disjunctive syllogism, in spite of the fact that they can't give an uncontroversial example of a disjunctive syllogism with true premises and a false conclusion. There is no alternative logic of which I'm aware that is not inconsistent with some plausible principle of implication. Expunging counter-intuitive results from the logic of implication is like trying to slay the hydra; as means are found to remove one awkward result ten more arise in its place.

A second problem with existing alternatives to classical logic is their difficulty. With few exceptions, the non-classical logics have been the concern of trained mathematical logicians. The alternatives are often presented as axiom systems, facility with which does not come easily to those without a mathematical bent; formulas are often perversely long and difficult to give intuitive sense to. Moreover, the alternatives generally employ formal devices (like characteristic matrices), which present additional technical challenges. The alternative logics are difficult in a further sense, viz., they are difficult to make sense of. For example, the alternatives often involve semantic distinctions (like extensional vs. intensional senses of "and" and

“or”) which are not easy to explicate, as critics of the relevance logics like to point out (e.g., Burgess 1983, 47ff.). The non-classical systems are interesting to study both in their own right and with respect to one another. But studying non-classical logics leaves the impression that much work needs to be done before we can turn attention to ordinary argumentation. What is wanted is a set of rules sufficiently rooted in discursive practice that competent speakers can apply them without lengthy preparatory studies and without having to master concepts that are less than perspicuous.

A further indication of the unsuitability of non-classical logics is the absence of non-classical texts devoted to ordinary argumentation. Instead, existing non-classical texts teach the formalism in the context of the mathematical development of a theory. There are a few texts on logic and philosophy which present modal logic in the context of philosophical problems, such as certain proofs of God’s existence (e.g., Purtill 1989, Bradley and Swartz 1979). Such texts present modal logic in connection with problems in metaphysics and epistemology, but they still lack applications of the formalism to arguments from ordinary discourse. Indeed, Bradley and Swartz grant that the concepts of classical propositional logic “play a more obvious role in ordinary argumentation and inference than do modal ... concepts” (219). Thus, it is not surprising that these texts present logics that are *extensions* of classical logic; additional devices make sense in the context of working with special kinds of arguments. But so far there are *no* textbook presentations of logics which are genuine *alternatives* to classical logic (e.g., Anderson and Belnap’s entailment). Until the appearance of a text that applies an alternative logic *systematically* to ordinary argumentation, they are *prima facie* unlikely to advance the cause of informal logic.

I am not suggesting that classical logic texts all work with ordinary arguments. Far too many concern themselves with examples like

The moon is made of green cheese, and the planet Mars is made of milk chocolate.

∴ The moon is made of green cheese. (Hausman et al. 2007, 9)²

But texts like Pospesel 2000, which draws the bulk of its illustrations and problems from ordinary discourse, demonstrate that classical propositional logic *can* be useful for analyzing a variety of everyday arguments. The success of a text like Pospesel 2000 suggests that as long as informal logic is going to include some formal logic, it ought to be a system as easy to use as Pospesel’s.

Likewise, any formal system employed in informal logic ought to capture our logical intuitions at least as well as Pospesel’s. In constructing PL– I sought to provide resources sufficient to demonstrate the intuitively valid natural language arguments (i.e., those lifted from newspapers, etc.) in Pospesel 2000. But even though classical logic squares with our intuitions about Pospesel’s natural language arguments, there are still elements that do not fit with discursive practice. There are the paradoxes, of course, but underlying the paradoxes are more fundamental

conflicts with discursive practice, conflicts present in *both* classical and non-classical logic. These conflicts create pedagogical problems more difficult to surmount than any of the technical challenges presented by the non-classical logics. PL– avoids these pedagogical problems by focusing upon an aspect of discursive practice that interests informal logicians rather more than formal ones – the *purpose* for which an argument is given.

2. Argument

Whereas informal logicians conceive of an argument as a discourse whose purpose is to justify or explain, formal logicians (classical and non-classical alike) embrace a more abstract conception, viz., “a list of statements, one of which is designated as the conclusion and the rest of which are designated as premises” (Skyrms 1986, 1-2). Thus, they depart from discursive practice early in the game. To prepare the way for PL–, I show first that the abstract conception—rather than the material conditional—underwrites paradoxes like (1), above, and (2).

(2) I’m not in Arizona.

∴ If I’m in Arizona, I’m in New York.

Ostensibly, formal logicians are concerned with argument as it occurs in discursive practice.

Typically, an argument consists of certain statements or propositions, called its premisses, from which a certain other statement or proposition, called its conclusion, *is claimed to follow*. ... When an argument is used seriously by someone (and not, for example, just cited as an illustration), that person is asserting the premisses to be true and also asserting the conclusion to be true *on the strength of* the premisses. (Lemmon 1978, 1) (my italics)

Yet they allow the abstract conception of argument to trump the discursive one. For, if they focused on sets of statements, one of which is claimed true on the strength of the rest, they would balk at

(3) Boris Yeltsin is an American and he is also a poet.

Therefore, he is a poet,

and

(4) I’m not buying you a Honda.

Therefore, I will either buy you the most powerful Honda made or no Honda at all

instead of presenting them as *paradigm* valid arguments (Pospessel 2000, 4 and 95).³ (3) is not part of discursive practice because it is a circular argument; it stands to arguments as imitation pearls to pearls. (4) offends discursive practice by introducing content in the conclusion that occurs nowhere in the premisses. (3) and (4) do, however, point to necessary conditions for any set of statements to count as an argument in the concrete sense: (i) the conclusion may not be asserted in the premisses, and (ii) the contents of the conclusion must occur somewhere in

the premises. Thus, notice that the familiar syllogisms of propositional logic (modus ponens, etc.) all satisfy these criteria; carefully formulated, the schemes for conditional proof and *reductio ad absurdum* will too.

It is both ironic and revealing that (3) and (4) occur in a text that strives to connect formal logic to discursive practice. Unlike the other valid patterns Pospesel introduces, none of these is excerpted from a justificatory or explanatory discourse. They are artifacts the author hopes will lend an air of naturalness to simplification and addition. Pospesel does attempt to defend (4) as an instance of discursive practice.

Nevertheless, we do occasionally reason in this way. My 14-year-old brother-in-law was bugging his father for a lightweight Honda motorcycle. My father-in-law told him, "I will either buy you the most powerful Honda made or no Honda at all." Had my father-in-law been asked to justify his statement, I'm confident he would have replied, "I'm not buying him a Honda." The argument 'I'm not buying him a Honda; therefore, I'll either buy him the most powerful Honda or no Honda' is, of course, an inference sanctioned by the Wedge In Rule. (ibid.)

This account won't do, however, for there is a more plausible interpretation of the father-in-law's disjunction: It's an elliptical way of saying, "I'm not buying him a Honda," by using a disjunctive syllogism with the obvious unstated assumption, "I'm not buying him the most powerful Honda."

(3) and (4) are not attempts at justification or explanation; rather, they are entailments. Entailment is an abstract counterpart of justification and explanation. It is what remains of a good argument when we abstract from its justificatory or explanatory purpose, viz., a conclusion which will be true as long as its premises are. The inference rules (or axioms) of *both* classical *and* non-classical logic are attempts to characterize valid arguments in the abstract sense—a set of statements, one of which is *entailed* by the rest. Logics are deemed more or less successful depending on one's intuitions about entailment; the proliferation of logics in the past century betrays tremendous diversity in these intuitions. There is disagreement even about (3) and (4),⁴ let alone about cases like P&-P entails Q. Entailment is, of course, part of discursive practice; the premises of a disjunctive syllogism entail its conclusion. Controversy arises when we attempt to extend entailment to cases which play no role in everyday practices of justification and explanation. Lacking such a role, the diversity of intuition concerning entailment is inevitable; for then intuitions are generated by less than decisive analogies.

There *is* agreement of intuitions—and not just among logicians—concerning the traditional propositional syllogisms.⁵ And, not surprisingly, they play a role in everyday practices of justification and explanation. We want the inference patterns that constitute PL– to be part of discursive practice, so the traditional propositional syllogisms will be the core of the system. But this means that PL– will not be a logic of entailment in any interesting sense. Rather it will be an attempt to characterize good arguments in the concrete sense: *a set of statements one of which is justified or explained by the rest.*

Before proceeding, it is crucial to understand that I am *not* claiming (3) or (4) are invalid rather than valid arguments. I am even willing to grant that that P&Q entails P. I deny only that the premise P&Q justifies or explains the conclusion P; that is, I deny that such entailments are arguments. (3) and (4) are dressed with “therefore” to *look like* concrete arguments, but they are not. They are, of course, valid arguments in the abstract sense, but I am putting that concern aside in no small part because simplification and addition give rise to the paradoxes.

Perhaps this is not obvious. After all, the paradoxes are demonstrable with truth tables, and texts often justify argument forms by appeal to truth tables (e.g., Pospesel 2000, 155; Lemmon 1978, 22). Against this suggestion I urge the priority of argument forms over semantics. Historically propositional logic emerged in Stoic thought as a codification of discursive practice. The priority of argument forms was clear to the Stoics, who, notoriously, *disagreed* over the semantic analysis of the statement connectives while *agreeing* on the indemonstrable forms containing those connectives (Mates 1961, 42ff.). The semantic analyses of statement connectives still rest upon forms embedded in discursive practice. For example, the most plausible defense of the truth table for the conditional consists in arguing that *if* the conditional is to be treated truth-functionally and *if* we are to preserve intuitions about modus ponens, modus tollens and denying the antecedent, then there is no choice but to accept that a conditional is true if its antecedent is false or its consequent true (Rubin and Young 1983, 92-4). In contrast, it’s hopeless to support truth tables by appeal to semantic intuitions. As fast as instructors point to conditionals that illustrate the truth table for “if P, then Q,” for instance, undergraduates find conflicting cases. This is typical of the pedagogical difficulties I mentioned above: Students feel they’re being tricked when instructors insist that an analogy with one example (and usually a peculiar one like “if that’s a good argument, then I’m a monkey’s uncle”) should settle the matter. In any event, if blame for the paradoxes can be laid at the feet of simplification and addition, there should be no surprise that systems based on semantic analyses yield the same results.

Consider, then, natural deduction systems like Lemmon 1978 and Pospesel 2000, which aim to capture classical propositional logic in an *intuitive* manner. These systems rely primarily upon argument forms from discursive practice; simplification and addition are exceptions. Why do systems which prize discursive intuitions give counterintuitive results? It is *not* because discursive practice involves inconsistencies. Rather, it is because simplification and addition, which are *not* part of discursive practice, permit unnatural cases of assumption dependence in conditional proofs, which *are*, otherwise, part of discursive practice.

Some versions of conditional proof permit vacuous discharge of assumptions, a move that is clearly absent from discursive practice. Vacuous discharge is illustrated in the following proof of (1) above (cf. Mates 1972, 99).

{1}	1. P	Premise
{1}	2. $Q \rightarrow P$	1 Conditional Proof
{ }	3. $P \rightarrow (Q \rightarrow P)$	1, 2 Conditional Proof

In both lines 2 and 3 a conditional is derived. 3 is the more usual case, since 1, the line number corresponding to the antecedent of $P \rightarrow (Q \rightarrow P)$, is discharged from the assumption dependence column. Line 2 is a case of vacuous discharge, since the antecedent of 2 does *not* occur as a previous line and so there is nothing to discharge from the assumption dependence column. This move recalls addition, which allows introduction of a proposition with no prior occurrence. Neither Pospesel nor Lemmon allow vacuous discharge; they insist on more intuitive means for obtaining Mates's result. In Lemmon's system, for example, applications of conditional proof require that the consequent of a conditional derived by CP *depend on* its antecedent (15). Mates fails this requirement because the consequent of line 2 isn't derived from Q. Lemmon's restriction is imposed in the spirit of discursive practice; he aims to model suppositional reasoning, not to shorten proofs. Why, then, is he committed to the paradoxes?

Lemmon offers two demonstrations. The first adds a step of conditionalization to his proof of sequent 50, $P | \neg Q \rightarrow P$ (59).

{1}	1. P	Assumption
{1}	2. $\sim Q \vee P$	1 Addition
{ }	3. $Q \rightarrow P$	2 Sequent Intro, (Seq. 48, $\sim P \vee Q \neg P \rightarrow Q$)
{ }	4. $P \rightarrow (Q \rightarrow P)$	1,3 Conditional Proof

This proof is not modeled on discursive practice, since addition allows the introduction of Q to line 2 in spite of the fact that it has no prior occurrence. Lemmon justifies addition with a truth-functional analysis of disjunction (22). However, that analysis is no less problematic than the truth-functional analysis of conditionals, since in practice disjuncts stand in some logical connection with one another. Moreover, if truth-functional analyses warrant rules of natural deduction, then it's difficult to understand scruples about vacuous discharge.

Lemmon counters that even without addition the paradoxes are unavoidable.

Anyway, 50 and 51 can be proved using only the rules A, &I, &E, RAA, DN and CP, in each case in nine lines; it is an instructive exercise to discover these 'independent' proofs, since they reveal how difficult it is to 'escape' the paradoxes. (61)

How do these rules warrant content in a conclusion that occurs nowhere in the premises? Consider the 'addition free' proof of sequent 50.⁶

{1}	1. P	Assumption
{2}	2. Q	Assumption
{3}	3. $\sim P$	Assumption
{2,3}	4. $Q \& \sim P$	2,3 &I

{2,3}	5. $\sim P$	4&E
{1,2,3}	6. $P \& \sim P$	1,5 &I
{1,2}	7. $\sim \sim P$	3,6RAA
{1,2}	8. P	7DN
{1}	9. $Q \rightarrow P$	2,8CP

The proof rests on what I call ‘conjunctive subterfuge’. Compare lines 3 and 5. In 3 $\sim P$ depends innocuously upon itself, but in 5 it depends upon both itself and line 2, thanks to $\sim P$ and Q being conjoined in 4 and separated in 5. This subterfuge ‘justifies’ the claim that $\sim P$ depends on assumption Q , and so satisfies Lemmon’s scruple that the consequent of the conditional proved *depend on* its antecedent. But it does not depend on Q in the usual sense that our willingness to accept $\sim P$ at line 5 depends on our willingness to accept Q at line 2; that is, $\sim P$ does not depend on Q in the sense that calling Q into question would *ipso facto* call $\sim P$ into question. Apparently Lemmon’s concern for discursive practice is trumped by his commitment to the abstract conception of argument.

By shifting our attention to the discursive sense of “argument,” it’s easy to see how the paradoxes are generated. Within the discursive conception of argument a conclusion depends on an assumption when that assumption is used to *justify* that conclusion. Since Q in line 2 is not used in justifying $\sim P$ in line 5, $\sim P$ does not depend—in the discursive sense—upon Q . Thus, instead of being uncomfortable consequences of discursive practice, as Lemmon hints, proofs of the paradoxes are possible only by introducing maneuvers that are not part of discursive practice.

As indicated above, my real concern is not with the classical logic’s paradoxes *per se*, but with the conceptual maneuvers which make them possible, in particular, treating simplification and addition as though they are forms of argument. Such maneuvers leave students with the impression that logic relies on tricks which they’ll have to master just long enough to pass the course. For students who don’t enjoy symbolic logic for its own sake, the tricks of formal logic make it appear as a symbol game which, apparently, interests the teacher but has no claim upon them. *This* is the fundamental motivation for developing a different system of formal logic. Moreover, this criticism can’t be met by arguing, as Pospesel does, that moves like conjunctive subterfuge are only used in cases that are “invented by some logician to illustrate a defect in propositional logic” (231). Pospesel needs conjunctive subterfuge for his justification of disjunctive syllogism (130). It’s dangerous pedagogy that expects students to justify an unquestionable move from discursive practice by means of a peculiar symbolic manipulation.

3. PL–

PL– consists mostly of traditional syllogisms. In preliminary form these are

Modus Ponens (MP)	$P \rightarrow Q, P \therefore Q$
Modus Tollens (MT)	$P \rightarrow Q, \sim Q \therefore \sim P$

Disjunctive Syllogism (DS)	$P \vee Q, \sim P \therefore Q$ $P \vee Q, \sim Q \therefore P$
Conjunctive Syllogism (CS)	$\sim(P \& Q), P \therefore \sim Q$ $\sim(P \& Q), Q \therefore \sim P$
Constructive Dilemma (CD)	$P \vee Q, P \rightarrow R, Q \rightarrow S \therefore R \vee S.$

In each case the conclusion is not asserted in the premises but the content of the conclusion does occur in the premises. Each is an elementary form of justification or explanation; moreover, their conclusions depend on their premises in the sense that questioning a premise *ipso facto* questions the conclusion.

Besides these forms, PL– includes two argument schemes, whose conclusions are inferred from a derivation. The first scheme is conditional proof.

Conditional Proof (CP) Given a set of assumptions A_k ⁷ and a provisional assumption P, if Q is derivable from $A_k \cup P$, then from A_k derive $P \rightarrow Q$.

A conclusion inferred in accordance with this scheme depends on A_k . The following proof of hypothetical syllogism illustrates CP.

{1}	1. $P \rightarrow Q$	Assumption (A)
{2}	2. $Q \rightarrow R$	A
{3}	3. P	Provisional Assumption (PA)
{1,3}	4. Q	1,3 MP
{1,2,3}	5. R	2,4 MP
{1,2}	6. $P \rightarrow R$	3-5 CP

Since PL– contains only patterns from discursive practice, the consequent of a conditional proof depends on its antecedent in the robust sense that a criticism of the antecedent is *ipso facto* a criticism of the consequent. The second scheme is *reductio ad absurdum*.

Reductio ad Absurdum (RAA) Given a set of assumptions A_k and a provisional assumption P, if there are two sets S and S' such that $S \cup S' = A_k \cup P$, and (i) Q is derivable from S and (ii) Q contradicts S' or a statement derivable from S', then from A_k derive $\sim P$.

Here is an illustration.

{1}	1. $P \rightarrow (Q \vee R)$	A
{2}	2. $\sim Q$	A
{3}	3. $\sim R$	A
{4}	4. P	PA
{1,4}	5. $Q \vee R$	1,4 MP
{1,2,4}	6. R	5,2 DS
{1,2,3}	7. $\sim P$	4-6,3 RAA

I include RAA on the grounds that it is essential to historically important reasoning like the proof of the incommensurability of side and diagonal. But I have reservations.

The fact that a proposition derived from a provisional assumption, say P, conflicts with an otherwise unused member of the original premise set, say $\sim R$, is a less than robust case of a $\sim R$ being used in a derivation of $\sim P$. For most purposes it suffices to treat *reductio* proofs as the derivation of a known falsehood from a provisional assumption; from there, via modus tollens, the desired result can be obtained.

PL- has no forms governing biconditional statements since they can be paraphrased as pairs of conditionals. And unlike Stoic logic, there are no forms governing exclusive disjunctions; they can be paraphrased as an inclusive disjunction and a negated conjunction. Outside of avoiding extra operators, paraphrase may appear to have only pedagogical value: One paraphrases a premise or conclusion to make the transition to the symbolic formula more transparent. This impression is reinforced by translation exercises involving isolated statements. However, when the statements to be translated occur in the context of an argument, paraphrase can play a more important role (Sherry 1991). Observe how paraphrase is employed in the following.

If the creation story is a true literal description (C), then for the first three days of the earth's existence there was no sun (S). The concept of 'day' is defined by reference to the sun (D). It cannot both be the case that the concept is so defined and that the earth existed three days before the sun was created. From this it follows that the creation story in Genesis is not a true literal description (Pospesel 1971, 29).

The argument is simply demonstrated in classical logic or PL-.

- | | |
|----------------------|--------|
| 1. $C \rightarrow S$ | A |
| 2. D | A |
| 3. $\sim(S \& D)$ | A |
| 4. $\sim S$ | 3,2 CS |
| 5. $\sim C$ | 1,4 MT |

But its paraphrase is not as straightforward as translating simple statements to statement letters and natural language operators to symbolic operators. The component clauses,

for the first three days of the earth's existence there was no sun

and

the earth existed three days before the sun was created,

would not both be rendered by S were they to be symbolized in isolation; the first is a negation while the second is simple. In spite of the different forms in the original clauses, it's necessary to paraphrase both clauses identically to bring the original argument within the scope of propositional logic. This example undermines the idea that statements exhibit a unique propositional form. Form is dictated not simply by the pattern of simple statements and operators, but also by the *use* to which a statement is put in an argument. This example also reminds us that successful paraphrase presupposes that one has in mind the forms of valid argument.

The preceding remarks underlie my treatment of negation. In the face of arguments involving statements used to contradict one another, it is immaterial which is represented as a negation. Thus,

You failed the course only if you failed the final.

You passed the final.

∴ You passed the course.

can be represented variously as

$C \rightarrow F$

$\sim F$

∴ $\sim C$,

or

$\sim C \rightarrow F$

$\sim F$

∴ C ,

etc. Moreover, because we choose arbitrarily which statement to represent as a negation, PL₋ regards each of these symbolic arguments as an instance of modus tollens. Analogously,

$\sim P \vee Q$

P

∴ Q , etc.

and

$\sim(P \& \sim Q)$

$\sim Q$

∴ $\sim P$, etc.

are instances of disjunctive and conjunctive syllogism, respectively. This policy imitates discursive practice and enables PL₋ to avoid double negation, which may not.

The absence of addition from PL₋ is no cause for alarm because CD warrants disjunction introduction. Nor is the absence of conjunction, since it is a derived rule of PL₋, like hypothetical syllogism.⁸ But simplification is not so easily dismissed. Conjunctions *do* occur in discursive practice, in conjunctive syllogisms and as components of conditionals, disjunctions and negations. Why should discursive practice lack the resources for eliminating “and”?

One might claim that such inferences are too simple to employ consciously in ordinary contexts (cf. Pospesil 2000, 27). But this would explain the anomaly only with an unwarranted presumption. Does inferring

We’re taking the station wagon.

from

We’re taking the dog and the kids. If we take the dog, then we take the station wagon or the truck. We don’t take the truck if we take the kids.

require detaching (unconsciously, via simplification) “we take the dog” and later “we take the kids” from the first premise? Compare this with a case in which detachment is logically necessary. In inferring

We’re taking the pickup.

from

We’re taking Amy. We won’t take Amy unless we take the dog. If we take the dog, then we take the pickup,

we can’t draw the conclusion until we detach “we take the dog,” via, say, disjunctive syllogism. In this argument “we take the dog” is not asserted in any of the premises, and we can’t infer “we’ll need the station wagon” unless we are *warranted* in asserting, “we take the dog.” We obtain that warrant by detaching “we take the dog.” There’s one sense in which detachment in the station wagon argument is simpler than detachment in the pickup argument: Detachment via disjunctive syllogism requires a premise *in addition to* the disjunction “we can’t take Amy unless we take the dog.” On the other hand, the conjunctive premise alone suffices for detachment by simplification. But is it necessary even to detach a conjunct before using it? In the station wagon argument “we take the dog” is *already* asserted in the premises, so there is no need to detach it in order to obtain a warrant. Thus, the purpose served by detaching a conjunct is not the purpose served by detaching a statement via syllogism, viz., obtaining a warrant.

The rationale is, rather, the *presumption* that “and” is an operator on a par with “if, then,” “or” and “not.” By analogy, we require rules for using conjunctive premises and deriving conjunctive conclusions (cf. Lemmon 1978, 19). But this is a strained analogy. Statement operators are functions from a statement (or pair of statements) to a further statement, which asserts something different from the original statement(s). For instance, asserting a logical (or causal) relation between the components of a conditional is a different matter from asserting the components. In such cases, generating compound statements and accessing their unasserted contents is possible only in the company of *additional* statements. Taking account of these additional statements requires rules for introducing and eliminating “→,” “∨” and “~.” But there is no such rationale for simplification and conjunction. Their value consists simply in enabling logical theory to treat uniformly statements that are *grammatically* compound.

Of course, conjunctions are asserted in arguments, but they are not used in the manner of conditionals, disjunctions and negations. A conjunction calls attention to a set of statements that share a common theme (cf. Rundle 1983), but unlike other compounds, a conjunction does *not* assert a logical or causal relation between those statements.⁹ How, then, shall we understand the role of conjunctions in conditionals, disjunctions and negations?

The proper conception requires widening the assumption that “if, then,” “or” and “not” are functions from statements or pairs of statements to a further statement. These operators *are* used to assert relations among statements, but more generally

they are used to assert relations between *sets* of statements. Thus the general forms of conditionals, disjunctions and negations are $P_i \rightarrow Q_j$, $P_i \vee Q_j$, and $\sim P_i$, respectively. The general forms necessitate reformulating the rules of PL-. If P_{i^*} is a subset of the set of statements P_i , and P_{i^*}' its complement in P_i , we have

Modus Ponens (MP)	$P_i \rightarrow Q_j$	
	$P_i / \therefore \rightarrow Q_j$	
Modus Tollens (MT)	$P \rightarrow Q$	
	$\sim Q_{j^*} / \therefore \sim P_i$	
Disjunctive Syllogism (DS)	$P_i \vee Q_j$	$P_i \vee Q_j$
	$\sim P_{i^*} / \therefore Q_j$	$\sim Q_{j^*} / \therefore P_i$
Conjunctive Syllogism (CS)	$\sim P_i$	
	$P_{i^*} / \therefore \sim P_{i^*}'$	
Constructive Dilemma (CD)	$P_i \vee Q_j$	
	$P_i \rightarrow R_k$	
	$Q_j \rightarrow S_m / \therefore R_k \vee S_m$	

We needn't worry that the premise forms in our rules apparently require us to form conjunctions. For, we can allow the elements of a set of statements to occur on different lines in a proof, as in the following demonstration (cf. Pospesel 2000, 27).

- 1. K A
- 2. O A
- 3. (K,O)→B A
- 4. B 3,1,2 MP

Anyone finds this expedient objectionable may appeal to the derived rule $P, Q \vdash (P,Q)$ (see note 8 above) to form conjunctions. Similarly, there is no need for conjunction to form the conclusions of arguments like Harvey's

In an hour, a human heart **THROWS** out more blood than the human's own weight. If this is so and if blood flows only **OUTWARD** from the heart, then the heart creates **MORE** blood in an hour than the weight of a human. But the heart cannot do this. If the blood does not flow only out of the heart, then it must **CIRCULATE** through the body and **REENTER** the heart. Thus, the view that blood flows only out of the heart is false and the view that blood circulates through the body and reenters the heart is true. (Pospesel 1971, 53-4),

as in the following classical demonstration.

- 1. T A
- 2. (T&O)→M A
- 3. $\sim M$ A
- 4. $\sim O \rightarrow (C \& R)$ A $\therefore \sim O \& (C \& R)$
- 5. $\sim (T \& O)$ 2,3 Modus Tollens

- | | |
|-------------------------|---------------------------|
| 6. $\sim O$ | 5,1 Conjunctive Syllogism |
| 7. C&R | 4,6 Modus Ponens |
| 8. $\sim O \& (C \& R)$ | 6,7 Conjunction |

Once the members of a set of statements have been derived, gathering them on a single line provides no further justification or explanation. For PL $^-$, then, the demonstration is finished at line 7. The proof through line 7 is just an abbreviated way of treating Harvey's reasoning as a pair of arguments. But here too, one may invoke the derived rule $P, Q \vdash (P, Q)$ to obtain line 8. Finally, generalized CP is formulated so as to demonstrate a conditional with a conjunctive consequent without a transformation to form conjunctions.

Conditional Proof (CP) Given a set of assumptions A_k and a provisional assumption P_i , if there are sets S^m such that $\cup S^m = A_k \cup P_i$, and (i) each element of Q_j is derivable from some S^j , and (ii) each S^m occurs in at least one such derivation, then from A_k derive $P_i \rightarrow Q_j$.

Again, $P, Q \vdash (P, Q)$ could be invoked to justify this formulation of CP. Before we illustrate this rule, a final refinement to PL $^-$ is necessary.

Treating a conjunction as a set of statements rather than a single, compound statement cannot avoid entirely *dismantling* a set of statements. An analogue to simplification is necessary to track the use of provisional assumptions in derivations by CP. Recall the station wagon argument from a few paragraphs back. One of its premises is a conjunction, "we're taking the dog and the kids." By treating the conjuncts as separate premises we can represent this conjunction in a way that avoids formally dismantling the conjunction.

- | | | |
|-------------------------------|---|----------------|
| 1. D | A | |
| 2. K | A | |
| 3. $D \rightarrow (S \vee T)$ | A | |
| 4. $K \rightarrow \sim T$ | A | $\therefore S$ |

This representation is justified because both conjuncts are asserted and the generalized forms are indifferent to D and K occurring on separate lines. However, if the conjunction occurs unasserted (as in "if we go for a week, then we'll take the dog and the kids"), when detached, it yields a set of statements on a single line, as in line 5 below.

- | | | |
|-------------------------------|--------|----------------|
| 1. $W \rightarrow (D, K)$ | A | |
| 2. W | A | |
| 3. $D \rightarrow (S \vee T)$ | A | |
| 4. $K \rightarrow \sim T$ | A | $\therefore S$ |
| 5. D, K | 1,2 MP | |

Thus far we lack the resources to continue this derivation by detaching $S \vee T$ and $\sim T$.

We could solve this problem *without* formally dismantling the conjunction by detaching $S \vee T$ and $\sim T$ in accordance with a version of modus ponens that includes irrelevant information,

$$P_i \rightarrow Q_j, P_i, R_k \therefore Q_j.$$

However, this rule—which is presumed by any logic of entailment—invites the fallacy of strengthened antecedent. Thus,

If my daughter majors in philosophy, I'll be delighted. So, if my daughter majors in philosophy and fails all her classes, I'll be delighted,

which is classically valid, would also be valid in PL— under the proposed rule.

{1}	1. $P \rightarrow D$	A
{2}	2. P, F	PA
{1,2}	3. D	1,2 MP
{1}	4. $(P, F) \rightarrow D$	2-4 CP

By avoiding, once again, premises that are not really used in a derivation, we can avoid the fallacy of strengthened antecedent.

This requires isolating the conjunct(s) actually employed in deriving a conditional's consequent; thus, we introduce

Tracking (T) $P_i \therefore P_{i^*}.$

To promote logical hygiene, we apply **T** *just in case* a proper subset of P_i is to be used in one of the syllogisms. The usual rule for assumption dependence applies to **T** unless P_i is provisionally assumed. If P_i is provisionally assumed on line m , and $i=1,2,\dots,n$, then m_1, m_2, \dots, m_n appear in the assumption dependence column on line m . Further, when **T** is applied to yield P_{i^*} , the corresponding values m_i appear in the assumption dependence column. Conditionalization on a provisional assumption P_i is not allowed unless the line number of *each* member of P_i appears in the assumption dependence column of a derived formula Q . The assumption dependence rule governing **T** avoids the fallacy of strengthened antecedent.

{1}	1. $P \rightarrow D$	A
{2 ₁ , 2 ₂ }	2. P, F	PA
{2 ₁ }	3. P	2 T
{1, 2 ₁ }	4. D	1,3 MP
{1}	5. $(P, F) \rightarrow D$	2-4 CP (wrong)

Line 5 is illegitimate because 2₂ does not occur in the assumption dependence column in line 4.

With **T** to keep track of dependence relations, we can illustrate generalized CP.

{1}	1. $P \rightarrow Q$	A
{2}	2. $R \rightarrow S$	A
{3}	3. $\sim(Q, S, T)$	A
{4 ₁ , 4 ₂ }	4. P, R	PA
{4 ₁ }	5. P	4 T(6)

{1,4 ₁ }	6. Q	1,5 MP
{1,3,4 ₁ }	7. $\sim(S,T)$	3,6 CS
{4 ₂ }	8. R	4 T(9)
{2,4 ₂ }	9. S	2,8 MP
{1,2,3,4 ₁ ,4 ₂ }	10. $\sim T$	7,9 CS
{1,2,3}	11. $(P,R)\rightarrow\sim T$	4-10 CP ¹⁰

Here conditionalization at line 11 is permitted because both 4₁ and 4₂ occur in the assumption dependence column for line 10. The justification for line 5 includes a parenthetical “6” to convey **T**’s auxiliary function. The rationale for separating P from (P,R) is to track its role in deriving line 10, and the parenthetical indicates exactly where P is used as one of the premises in a series of syllogisms that lead to 10. This convention prevents detaching a conjunct for its own sake by marking **T** as an auxiliary transformation rather than an argument form.

Although its statement is complicated, Generalized RAA presents no special problems.

Reductio ad Absurdum (RAA) Given a set of assumptions A_k and a provisional assumption P_i , if there is a set Q_j and sets S^m for which $\cup S^m = A_k \cup P_i$, and, if (i) each element of Q_j is derivable from some S^i and (ii) Q_j contradicts some S^i or a set of statements derivable from some S^i , and (iii) each S^m occurs in (i) or (ii), then from A_k derive $\sim P_i$.

Here is a simple illustration of generalized RAA.

{1}	1. $P\rightarrow Q$	A	$\therefore \sim(P,\sim Q)$
{2 ₁ ,2 ₂ }	2. $P,\sim Q$	PA	
{2 ₁ }	3. P	2, T(4)	
{1,2 ₁ }	4. Q	1,3 MP	
{2 ₂ }	5. $\sim Q$	2, T(6)	
{1}	6. $\sim(P,\sim Q)$	2-4,5 RAA ¹¹	

4. The Limits of PL– and their Significance

Consider any set of statements that can be constructed from simple statements and the operators “if...then,” “and,” “or,” and “not.” Classical propositional logic enables us to say about any such set—one member of which is designated as conclusion and the rest as premises—whether it constitutes a valid argument. PL– can make no such claim, in part because it doesn’t recognize many such sets as arguments. Consider, then, such sets as also satisfy minimal formal conditions for arguments: (i) the conclusion is not already asserted in the premises and (ii) the content of the conclusion is included in the premises. Among *those* arguments some are valid and some are invalid. Lacking a formal semantics, PL– has nothing to say about the invalid bunch¹² and again, lacking a formal semantics, PL– has no

means to argue that it is able to demonstrate the validity of *all* the members of the valid bunch. Can a set of formal procedures which is unable to deal with either of these issues wear the mantle of a logical system? This is a deep and difficult question to which I won't be able to do justice. But if PL₋ is worth teaching to students and worth exploring in its own right, some kind of response is necessary.

Classical propositional logic stems from Frege's *Begriffsschrift* (Frege 1879), but not until 1921 was its completeness established (Post 1921). Thus, given that Russell, Whitehead, and Lewis put classical propositional logic to good use *prior* to 1921, the lack of a completeness proof shouldn't, by itself, count against the systematic character of a set of inference rules. Moreover, completeness merely shows that a system's semantics matches its syntax; it says nothing about the match between a logical system and the set of arguments (or entailments, if one prefers) that it sanctions. The latter sort of completeness—which we can call *intuitive completeness*—is a goal of which classical propositional logic falls short: Witness the paradoxes. Intuitive completeness is the goal toward which PL₋ strives. A proof that PL₋ achieves this goal would establish its systematicity beyond a doubt, but it won't be found in this paper. For one thing we have given only necessary formal conditions for an argument constructed from simple statements and sentential operators. Such a demonstration would require a complete formal characterization of a propositional argument (i.e., necessary and jointly sufficient conditions). Only then could formal semantics be brought to bear without treating PL₋ as a logic of entailment. These are tasks for the future. Even if they prove to be vain hopes, however, that outcome need not undermine the suitability of PL₋ as a logic for informal logicians. As long as it satisfies the demand for intuitive completeness better than classical propositional logic, PL₋ is sufficiently systematic for the needs of informal logicians. There is a simple reason for this: Informal logicians who value an account of logical form (e.g., Hatcher) have generally appealed to classical logic. So how does PL₋ fare in the quest for intuitive completeness?

Prima facie it fares better than its classical cousin, for it avoids the two paradoxes and the fallacy of strengthened antecedent. Yet there are arguments that call this *prima facie* judgment into question. I shall argue that their existence does not undermine PL₋ sufficiently for informal logicians to prefer classical logic.

First is an alleged argument (in the concrete sense) that's intuitively valid, but indemonstrable in PL₋.

If Norma is offered either a FELLOWSHIP or a teaching ASSISTANTSHIP, she will do GRADUATE work. Therefore, she will do graduate work if she is offered a fellowship. (Pospesel 2000, 94)

This argument is demonstrable in classical logic.

- | | | | |
|-----|-------------------------------|-------------|------------------------------|
| {1} | 1. $(F \vee A) \rightarrow G$ | A | $\therefore F \rightarrow G$ |
| {2} | 2. F | PA | |
| {2} | 3. $F \vee A$ | 2, Addition | |

{1,2}	4. G	1,3 Modus Ponens
{1}	5. $F \rightarrow G$	2-4 Conditional Proof

Since **CD** is the only means by which **PL**– introduces a disjunction, it’s clear that this argument is not demonstrable in **PL**–. Against this criticism, I suggest we haven’t really an argument here. Unlike most Pospesel’s illustrations, this one is *not* paraphrased from an argument that occurred in print. It’s doubtful that anyone would advance the conclusion on the strength of the premise. For a natural paraphrase of the premise is

If Norma is offered a fellowship, she will do graduate work, and if she is offered a teaching assistantship, she will do graduate work.¹³

In view of this paraphrase, the conclusion merely rehearses material in the premise, and so fails to count as an argument. Of course, disjunctive antecedents *can* occur in genuine arguments, but we can deal with them in the manner of the preceding paraphrase.

The next example is intuitively invalid but demonstrable in **PL**– as well as classical logic.

It’s not the case that if the U.S. CAPTURES bin Laden, they END the terrorist threat. So, the U.S. captures bin Laden but they don’t end the terrorist threat.

Some authors refer to this form—which, to be sure, is absent from discursive practice—as the third paradox of material implication (e.g., Lycan 2001, 26). The argument is, regrettably, demonstrable in **PL**–.¹⁴

{1}	1. $\sim(C \rightarrow E)$	A	$\therefore C, \sim E$
{2}	2. $\sim(C, \sim E)$	PA	
{3}	3. C	PA	
{2,3}	4. E	2,3 CS	
{2}	5. $C \rightarrow E$	3-4 CP	
{ }	6. $\sim(C, \sim E) \rightarrow (C \rightarrow E)$	2-5 CP	
{1}	7. C, $\sim E$	1,6 MT	

Discursive practice explains how this inference goes wrong. Ordinarily when we deny a conditional we mean to make the weaker claim that it’s possible for the antecedent to be true while the consequent is false, *not* the stronger claim that the antecedent is true and the antecedent false. It’s plain why neither **PL**– nor classical propositional logic can deal well with denied conditionals: they require an additional operator to express that P and Q are consistent, say $P \boxplus Q$. A related difficulty shows up in negated conjunctions. Sometimes a negated conjunction denies that all the conjuncts in a conjunction are true; but sometimes a negated conjunction makes the stronger claim that a set of conjuncts is inconsistent. To distinguish the two cases we require something like \bullet ; thus, $\sim(P \bullet Q)$ is stronger than $\sim(P, Q)$. The paradox arises because the weaker $\sim(C, \sim E)$ would justify claiming $C \rightarrow E$, but the

denial of $C \rightarrow E$ can justify only $C \bullet \sim E$, not $C, \sim E$. This subtlety is passed over by the application of MT in line 7.

Like classical logic, PL₋ must adopt some stance toward the third paradox. The classical logician Quine, who scoffs at modal logic and, on occasion, ordinary language (Quine 1966, 144), treats such cases as ‘don’t cares’ (Quine 1960, 258–9). I prefer to excuse PL₋ on the grounds that negated conditionals are outside its scope.¹⁵ In order to avoid the problem PL₋ would have to introduce % and rules governing inferences in which it is involved. For example, it could be required that any appearance of a negated conditional be replaced by a statement of compossibility and that negated conjunctions be expressed with a modal operator when appropriate. I think PL₋ should reject such options. On the one hand, doing so avoids unnecessary complication, since these inferences are unlikely to occur in ordinary argumentation. On the other hand, adopting the proposed rules would undermine PL₋’s proscription of entailment relations that do not also serve as patterns of justification or explanation. Indeed, as observed earlier, the proliferation of modal logics results from controversies over entailment relations involving modality. For the sake of working a manageable structure into the flux of discursive practice, PL₋, in common with *any* formal logic, is doomed to slight at least some features of that practice.

Like the third paradox, the last example is equally a problem for classical logic and PL₋. It occurs in Galileo’s *Dialogue concerning Two Chief World Systems*.

If the earth rotates, then if a rock is dropped from a tower it will land hundreds of yards to the west of the tower. In fact, though, if a rock is dropped from a tower it will land at the foot of the tower. Therefore the earth doesn’t rotate. (Galilei 1962, 126)

Intuitively the argument is valid; Galileo certainly treats it as such. But it’s not demonstrable in PL₋, and it’s demonstrably invalid in classical logic. Moreover, it refutes the suggestion that classical logic only misfires on “arguments that are invented by some logician to illustrate a defect in propositional logic” (Pospessel 2000, 231). By adding an instance of Boethius’s law,

$$(P \rightarrow Q) \rightarrow \sim(P \rightarrow \sim Q),$$

a proof is possible. According to Cooper, this law is characteristic of the conditional as it occurs in ordinary discourse (Cooper 1968, 304–5). I shy away from Cooper’s suggestion both because it pushes us in the direction of an entailment logic and because it has counter-intuitive consequences (cf., Kneale 1957). I suggest we use the law simply as a guide to suppressed premises.

Interestingly, Pospessel brings the tower argument within the scope of classical logic by means of clever paraphrase:

If the earth rotates, then a rock dropped from a tower will land to the west of the tower. It is false that a rock dropped from a tower lands to the west of the tower. Therefore the earth doesn’t rotate.¹⁶

The creation story argument above is not the only case for which classical logicians

are prepared to sacrifice apparent logical form to salvage intuitions of validity. I would defend this approach as well. Neither classical logic nor PL₋ is intended to capture anything as grand as the deductive structure of a language. Thus appeals to clever paraphrase and scope are both inevitable and excusable.

5. Conclusion

The five argument forms MP, MT, DS, CS, CD, the argument schemes CP, RAA, and the auxiliary transformation T constitute PL₋. PL₋ aims, rather modestly, to demonstrate valid propositional arguments occurring in discursive practice. This aim suits informal logicians, who are concerned more with discovering order in the chaos of discursive practice than with investigating the entailment relation or creating a calculating device. My experience with the variety of natural language arguments in Pospel 2000 leads me to believe that PL₋ succeeds in this aim. And since it satisfies the demands of intuitive completeness better than classical propositional logic, I submit that PL₋ is the right formal logic for informal logicians. This is not to say that PL₋ ought to be part of everyone's training in informal logic. The first approximation, and carefully chosen examples and exercises, are sufficient for introductory purposes—for the sake of helping students to discern the forest in spite of the trees.¹⁷

Notes

¹ See Cooper 1968 for a hefty sampling of intuitively invalid arguments that are classically valid and intuitively valid arguments that are classically invalid.

² The first exercise in this book (pp. 3-4) includes several arguments from ordinary discourse, e.g., an argument from Thurgood Marshall opposing the death penalty. As is typical, though, such examples disappear after the introduction of the formal theory.

³ The same complaint could be lodged against conjunction (cf. Pospel 2000, 27). Curiously, though, it is derivable from patterns that are part of discursive practice. See below.

⁴ Thus, Nelson rejects (3) (1930, 448), and Parry's Analytic Implication rejects (4) (1989, 102).

⁵ In point of fact, the rejection of disjunctive syllogism by relevance logicians constitutes the exception that proves the rule. For they explicate disjunction in such a way that cases of disjunctive syllogism from discursive practice still turn out to be valid.

⁶ The same strategy yields sequent 51 ($\sim P \mid \sim P \rightarrow Q$).

⁷ A_k indicates a set of k statements A_1, \dots, A_k .

⁸	{1}	1. P	A
	{2}	2. Q	A
	{3}	3. $\sim(P \& Q)$	PA
	{1,3}	4. $\sim Q$	3,1 CS
	{1}	5. $\sim(P \& Q) \rightarrow \sim Q$	3-4 CP
	{1,2}	6. $(P \& Q)$	5,2 MT

To be sure, conjunction is not part of discursive practice. By invoking it we can simplify somewhat the rules of PL₋. See below.

⁹ Geach 1972 scoffs at attempts to segregate conjunctions from other compounds on the grounds that doing so fails "to take the unasserted occurrences of propositions into account" (14). On the contrary, embracing simplification and conjunction as elementary argument forms requires

overlooking the difference between asserted and unasserted occurrences of propositions.

¹⁰ $A_k=A_3=\{P\rightarrow Q, R\rightarrow S, \neg(Q,S,T)\}$; $P_1=P_2=\{P,R\}$; $S^1=A_3\cup P_2$; and $Q_j=\{\neg T\}$.

¹¹ $A_k=A_1=\{P\rightarrow Q\}$; $P_1=P_2=\{P,\neg Q\}$; $Q_j=Q_1=\{Q\}$; $S^m=S^1=\{P\rightarrow Q, P\}$ and $S^2=\{\neg Q\}$.

¹² One reason to think PL₋ doesn't reveal a system is that it has no techniques for demonstrating invalidity. However, logicians have been aware of fallacious propositional forms as long as they've been aware of valid propositional forms (Sextus Empiricus, *Adv. Math.* viii, 432-433). One studies formal logic, in part, because it makes it easier to distinguish valid inferences from temptingly similar invalid ones. Thus, four of the five basic syllogism in PL₋ have fallacious cousins: affirming the consequent (cp. MP), denying the antecedent (cp. MT), disjunctive fallacy (cp. DS) and conjunctive fallacy (cp. CS). That each constitutes a fallacy is part of discursive practice, and it is pedagogically natural to teach them in conjunction with the basic syllogisms. Is that enough? Classical propositional logic has a complete solution to the problem, viz., truth tables; although, the solution requires a truth-functional treatment of "if...then" and occasionally gives the wrong result (cf. Cooper 1968). PL₋ has no decision procedure, but by appealing to the four fallacies mentioned it can still handle the majority of invalid, natural language arguments in Pospel 2000. (Exceptions are ex. 20, p. 170; ex. 24, p. 171, and the illustration, p. 179.) Fallacies are, in any case, failures to live up to the norms of discursive practice, so it is not a fatal flaw that PL₋ cannot detect them all. A skeptic about a given argument would, I think, be justified in remaining agnostic about the argument in the absence of a demonstration in PL₋.

¹³ Pospel is not afraid to paraphrase an "or" by means of an "and." See Pospel 2000, 102 (exercise 10). McKay and van Inwagen 1977 give several examples of statements containing "or" with the force of "and," e.g., "either the well-ordering theorem or Zorn's lemma leads to the axiom of choice" (355).

¹⁴ In classical logic the first two paradoxes imply the third, and the third the first two. In PL₋, however, the argument from the third to the first two fails because the tracking rule won't allow the inference from $\neg(P\rightarrow Q)\rightarrow(P,\neg Q)$ to $\neg(P\rightarrow Q)\rightarrow P$ or to $\neg(P\rightarrow Q)\rightarrow\neg Q$.

¹⁵ Sherry 1999 argues that the scope of classical logic is governed by the following rule: The paraphrased premises should be equivalent to or weaker than the original premises and the paraphrased conclusion should be equivalent to or stronger than the original conclusion (328). This rule avoids the 3rd paradox for both classical logic and PL₋, but it must be invoked far more often by the classical logician (e.g., to avoid the first two paradoxes).

¹⁶ E-mail communication 10/3/02.

¹⁷ I received thoughtful criticism from Ian Dove, Howard Pospel, and Joseph Fulda.

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